Antiferromagnetism

At OK, two interpenetrating and identical sublattices of magnetic ions, each spontaneously magnetized to saturation in zero applied field but in opposite directions.

Paramagnetic

Antiferromagnetic

Ferromagnetic

Ferrimagnetic

No longer true! Exchange bias in GMR!! AFM is not directly technologically important, but the theory can be extended to explain ferrimagnetism (next lecture) which has many applications.
What do magnetization curves look like?

Dia-, Para-, + Antiferromagnets; -

no spontaneous $M$ (overall)

large applied field induces small $M$

**Temperature Dependence of $X$**

$X$ behaves like a PM above $T_N$, goes through a maximum at $T_N$ and decreases below $T_N$

$\frac{1}{X}$ is a straight line above $T_N$. The line extrapolates to a negative temperature at $\frac{1}{X} = 0$
Neutron Diffraction

⇒ conclusive evidence of AFM ordering

• neutrons have a magnetic moment \( \therefore \) are scattered by the electronic magnetic moment

Other advantages:

• neutron beam wavelength \( \approx \) atomic spacing at room temperature

\[ \lambda = \frac{\hbar}{P} ; \quad \frac{p^2}{2m} = 3kT \quad \Rightarrow \quad \lambda^2 = \frac{\hbar^2}{3mkT} \]

\[ \lambda_{20^\circ\text{C}} = 1.49\text{Å} \]
\[ \lambda_{50^\circ\text{C}} = 1.42\text{Å} \]

• neutron scattering amplitude varies in an irregular way with atomic \# (\( \approx \) X-ray scattering amplitude \( \propto Z \)) \( \therefore \) neutrons can distinguish, say Fe + Co

The neutron diffraction pattern is DIFFERENT above + below \( T_N \) because of the additional ORDERING present in the AFM phase.
Diffraeted neutrons obey the Bragg equation,
\[ n\lambda = 2d\sin\theta \]

Observed lines determined by Crystal Symmetry

eg. bcc lattice, (100) planes

\[ \Theta \{ \begin{array}{l} \text{Random arrangement of elements } C \text{ and } D \\ \text{or} \\ \text{Random spin directions on ions of element } C \end{array} \} \]

\[ \begin{array}{c} \text{Element } C \\ \text{Element } D \end{array} \} \]

\[ \text{or} \]

\[ \begin{array}{c} \text{Element } C, \text{ spin up} \\ \text{Element } C, \text{ spin down} \end{array} \]  

\( (100) \quad \text{intermediate} \)

(a) PARAMAGNETIC

(b) ANTFERROMAGNETIC

Fig. 5.11 Diffraction by (a) disordered and (b) ordered structures.

(1) + (3) are in phase (by Bragg condition)

BUT reflection from intermediate plane (2) is exactly out of phase

\[ \because \text{NO (100) line!} \]

gain (1) + (3) are in phase

This time, (2) scatters with a DIFFERENT AMPLITUDE... doesn't exactly cancel

\( \Rightarrow \text{(100) "superlattice" line} \)
Body-centered cubic unit cell showing phase relationship for reflected beams from (100) planes. Resultant intensity is zero.
In AFM phase, the magnetic moments on each [111] plane are aligned in the same direction, but they are in opposite directions on successive planes.
Analysis of fcc lattice \( \Rightarrow (hkl) \) lines only appear, if \( h, k + l \) are either all odd or all even.

![Graph showing neutron diffraction patterns for MnO below and above the spin-ordering temperature of 120 K.](image)

**Figure 20** Neutron diffraction patterns for MnO below and above the spin-ordering temperature of 120 K, after C. G. Shull, W. A. Strauser, and E. O. Wollan. The reflection indices are based on an 8.85 Å cell at 80 K and on a 4.43 Å cell at 293 K. At the higher temperature the Mn\(^{3+}\) ions are still magnetic, but they are no longer ordered.

Room Temperature: peaks at regular fcc cubic reflection positions

Low Temperature: all the room temperature peaks (since there is no structural phase transition) plus peaks which are at positions not allowed on the basis of the chemical unit cell. The additional peaks can be indexed using a magnetic cell twice as large as the chemical unit cell.
localized moment/Weiss theory

- works well - most AFMs are ionic compounds
- use to explain $\chi(T)$

experimentally

\[ \chi = \frac{C}{T + (-O)} \]

a curie-Weiss law with negative $O$

\(\text{Remember, } O \propto y \text{- the molecular field constant}\)

\[ \therefore \text{ negative } O \Rightarrow \text{ negative "exchange forces"} \]

\[ \Rightarrow \text{ moments anti-align!} \) \]
Theoretically

Simplest Case:

- Divide lattice into 2 identical sublattices, A and B.
- Only consider A-B interactions (ignore A-A and B-B).

↓

Two "Molecular Fields!"

\[ H^A = -\gamma M_B \quad H^B = -\gamma M_A \]

Minus \( \rightarrow \) molecular field is opposite to the magnetization of the other sublattice.

Above \( T_N \):

\[ \chi = \frac{M}{H} = \frac{C}{T} \quad \Rightarrow \quad M = \frac{HC}{T} \]

For sublattice A:

\[ M_A = C'(H-\gamma M_B) \]

For sublattice B:

\[ M_B = C'(H-\gamma M_A) \]

Solve for \( M = M_A + M_B = \frac{2C'H}{T+C'\gamma} \)

\[ \chi = \frac{2C'}{T+C'\gamma} = \frac{C}{T+\Theta} \]
Susceptibility Below $T_N$ - Two Limits

(i) $H \parallel M$

\[ M_A^s \uparrow \text{by } \Delta M_A \rightarrow M = M_A - M_B \]
\[ M_B^s \downarrow \text{by } \Delta M_B = |\Delta M_A| + |\Delta M_B| \]

BUT, $M_A^s + M_B^s$ are given by the Langerin (or Brillouin) function

\[ N M \times L(x) \quad \text{or} \quad N M \times B(x) \]

\[ M_A = M^s + |\Delta M_A| \quad \Delta M_A = \Delta \alpha \times \text{slope} \]
\[ \Delta \alpha = \frac{m}{kT} \Delta H = \frac{m}{kT} (H - \gamma |\Delta M_B|) \]

\[ \chi_{\parallel} = \frac{2 N m^2 B'(J, \alpha)}{2 kT + N m^2 \gamma B'(J, \alpha)} \]

(ii) $H \perp M$

$H$ rotates spins

$\uparrow M \rightarrow O$

creating $M$ in field direction,

$+H_0$ in opposite direction.

At equilibrium

\[ H_{ext} = H \text{ (molecular field)} \]
\[ = 2 x H_0 \sin \theta \]
\[ = 2 \gamma M^s \sin \theta \]
\[ H_{ext} = \gamma x M \text{ (mice M} \]

\[ \chi = \frac{H}{M} = \frac{1}{\gamma} \]

$\chi_{\parallel}$ constant below $T_N$!
This is what it looks like:

\[ \frac{x_T}{x_{T_N}} \]

\[ T/T_N \]

Fig. 5.7 Calculated thermal variation of the susceptibility of an antiferromagnetic near and below the Néel temperature \( T_N \). The curve for \( \chi_\parallel \) is calculated for \( J = 1 \).

\[ \chi_p \text{ is the susceptibility of a powdered sample, with no preferred orientation of the crystals.}: \text{ average over all possible orientations} \]

\[ \Rightarrow \chi_p = \chi_\parallel \langle \cos^2 \Theta \rangle + \chi_\perp \langle \sin^2 \Theta \rangle \]

\[ = \frac{1}{3} \chi_\parallel + \frac{2}{3} \chi_\perp \]
Some real data:

![Graph showing the thermal variation of the susceptibility of MnO powder.](image)

Fig. 5.8 Thermal variation of the susceptibility of MnO powder. Bizette, Squire, and Tsai [5.4].

"Le point de transition à de la susceptibilité magnétique du protoxyde de manganèse MnO" Compt. Rend. 207, 449 (1938).

(The shape of the curve depends on the magnitude of the applied field because of magnetic anisotropy—we will discuss that next week... )
Below $T_N$ each sublattice is spontaneously magnetized in zero applied field, by the molecular field created by the other sublattice.

Solve graphically (just as for FM's).

$\Rightarrow$ net spontaneous magnetization is zero.

**Fig. 5.4** Spontaneous magnetizations of the A and B sublattices at temperatures below $T_N$ (schematic).

At $T_N$

\[ M_A = -\frac{C}{T_N} M_B \quad (H = 0) \]

\[ = -\frac{C}{T_N} M_B \]

But \[ M_A = -M_B \]

\[ \therefore \quad 0 = T_N \]

\[ \therefore \text{The Néel temperature } \equiv \text{ the } 0 \text{ value obtained from the } 1/X \text{ plot. } \]

(In fact $0 > T_N$ in most cases.)
Why is 'J' Negative?

In MnO: 

**SUPEREXCHANGE**

Geometry:

\[
\begin{array}{c}
\text{Mn}^{2+} & \infty & \text{Mn}^{2+} \\
\bullet & \infty & \bullet \\
\end{array}
\]

Remember - Mn\(^{2+}\) has 5 3d\(e^-\), one per orbital spin parallel

1. Make 1st Mn\(^{2+}\) 'spin-up':

\[
\begin{array}{c}
\text{O}^{2-} \\
\uparrow \uparrow \uparrow \uparrow \uparrow \downarrow \\
\bullet \\
\text{Mn}^{2+} \\
\end{array}
\]

2. Only the down-spin O\(_2\)p\(e^-\) can hybridize onto the 1st Mn\(^{2+}\) ion

3. The OTHER O\(_2\)p\(e^-\) is spin-up

4. The 2nd Mn\(^{2+}\) must be spin down

\[\Rightarrow\text{ overall AFM alignment}\]

(not strictly a negative J !)
Indirect exchange interactions in metals -
the RKKY interaction

(Ruderman, Kittel, Kasuya + Yosida)

Figure 17 Magnetization of a free electron Fermi gas at $T = 0$ in neighborhood of a point magnetic moment at the origin $r = 0$, according to the RKKY theory. The horizontal axis is $2k_Fr$, where $k_F$ is the wavevector on the Fermi sphere. (de Gennes.)

(from Kittel: Intro to Solid State Physics)

- either FM or AFM ordering
- depends on functional form of exchange integral $J(k)$
Why are some transition metals (e.g., Cr) AFM?

Look at the Fermi surface (a sphere for free electrons, complicated for transition metals!)

(001) section of BCC Brillouin Zone showing Fermi surface X-section for Cr

When two rather flat surfaces are parallel, an oscillatory spin density with wavevector $q$ occurs spontaneously.
Exchange Anisotropy in single domain particles of Co/CoO

Fig. 11.25 Hysteresis loops measured at 77°K of oxide-coated cobalt particles. Loop (1) results from cooling in a 10 kOe field in the positive direction, and loop (2) from cooling in zero field. Meiklejohn and Bean [11.28]

Phys. Rev. 105, 904 (1957)

- zero-field cooled sample ⇒ normal hysteresis
- field cooled sample ⇒ hysteresis loop SHIFTED

Mechanism - exchange coupling between the spins of ferromagnetic Co and antiferromagnetic CoO at the interface between them.

Note that:
1. Hc is increased in field-cooled sample
2. Hc is different for decreasing + increasing field
a) Cool particles below \( T_n \) of CoO in a field - Co ions in oxide 1st layer are \( || \) those in metal (the exchange) \( H = 0 \), \( M = \uparrow \). This spin arrangement persists when field is removed.

b) reverse field - Co spins reverse; reversal of spins in oxide is resisted by strong crystal anisotropy of CoO.

c) remove field - 'up' spins in oxide at interface force the Co spins to reverse.

\( H = 0 \), \( M = \uparrow \).

\( \Rightarrow \) UNIDIRECTIONAL (NOT UNIAXIAL!)
ANISOTROPY!!

Fig. 11.27 Mechanism of the shifted loop in Co-CoO. Arrows represent spins on cobalt atoms or ions, and open circles are oxygen ions. Points (a), (b), and (c) on the loop correspond to the three states at the left. Graham [10.1]

(The role of field cooling is to give the specimen a single easy direction. If it is cooled in zero field, the exchange interaction occurs at all interfaces, giving a random distribution of easy directions.)

MODERN APPLICATION: - spin valve

[Diagram of FM Free, FM Pinned AFM]