Solution of the Schrödinger Equation for an $e^-$ Encountering a Potential Step

On the left ($x < 0$) since $V = 0$ we already know the solution: $\psi = Ae^{ikx} + Be^{-ikx}$

On the right ($x > 0$), $V$ is not zero, but it is constant; the S. eqn. can be written

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi = (E-V)\psi$$

-the same form as for free $e^-$ but with $E-V$.

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A solution is: $\psi = Ce^{ik'x}$ where $k' = \frac{\sqrt{2m(E-V)}}{\hbar}$

-a plane wave with less K.E. (at least if $E>V$)

-just like the classical case.

What if $E<V$? Then $k'$ is imaginary $= \pm ik'1$

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so the wavefunction $\psi = Ce^{-ik'1x}$

decays exponentially on encountering the barrier. This is called TUNNELING!

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Problem 8.6, p. 148 (without time dependence)