Wave Packets and Uncertainty

We can understand electrons as particles (with a well-defined position) by building a wave packet.

A wave packet is a sum of a large number of plane waves each with different \( k \): it does not have a well-defined momentum! (cf. a single plane wave - well defined momentum/ poorly defined position).

In fact, not at all.

So to make a \( \delta \) function we need an \( \infty \) # of plane waves, so

\[
\delta \text{ function: } \Delta z = 0, \quad \Delta k = \infty \\
\text{plane wave: } \Delta z = \infty, \quad \Delta k = 0
\]

In general, a wave packet has finite width; needs a finite number of plane waves. In fact, \( \Delta z \Delta k \approx 2\pi \), \( \Rightarrow \)

\[
\Delta p \Delta z \geq \hbar \quad \text{Heisenberg's Uncertainty Principle}
\]

Note that the group velocity, \( \frac{\partial \omega}{\partial k} = \frac{\partial}{\partial k} \left( \frac{E}{k} \right) = \frac{\partial}{\partial k} \left( \frac{t_0 k^2}{2m} \right) = \frac{k k}{m} = \frac{p}{m} \)

i.e. the e- velocity is \( \frac{k}{k} \), so the wavepacket spreads out (higher k e's go faster!)

i.e. an electron wave exhibits strong dispersion.