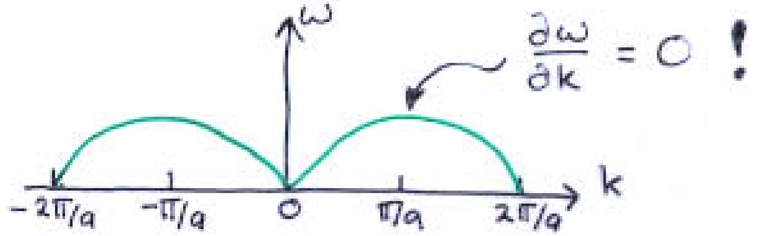
GROUP VELOCITY AND BRILLOUIN ZONES

When dispersion exists we define

phase velocity, $v_p = \frac{\omega}{k}$ — describes a pure wave with a single frequency

group velocity, $v_g = \frac{\partial \omega}{\partial k}$ — describes a wave pulse

If there's no dispersion, $\omega \propto k$, $\therefore \frac{\omega}{k} = \frac{\partial \omega}{\partial k}$ and $v_p = v_g$. BUT look at our 1D lattice:-



at $k = \pi/a$, $\lambda = 2a$, $v_g = 0$! We have a standing wave.
at $k = 2\pi/a$, $\lambda = a$ $\therefore$ all atomic displacements are identical. So $k = 2\pi/a$ is equivalent to $k = 0$! Other k values can be similarly shifted

$\therefore$ we can restrict our range of interest to $-\frac{\pi}{a} < k < \frac{\pi}{a}$ - this is the first Brillouin Zone.

PROBLEM 7.6, p.118