3-3 First Brillouin Zone

The dispersion curve in Fig. 3-5 contains some redundancy. To see this, consider a wave with \( k = -7\pi/4a \). We plot this wave in Fig. 3-9. If we compare Fig. 3-9 with Fig. 3-4, we see a striking similarity. Even though the wavelength is very different, the actual positions of the atoms are identical. In Fig. 3-10, we show the location of these two waves on the dispersion curve. We see that the wave numbers of these two waves differ by \( \Delta k = 2\pi/a \) and that they have the same frequency \( \omega \). The motion of the atoms for these two waves is identical. The wave numbers \( k = \pi/4a \) and \( k = -7\pi/4a \) describe the same wave. (They are mathematically different, but physically the same.) In general, any two wave numbers that differ by \( 2\pi/a \) describe the same physical wave in a one-dimensional lattice.

Fig. 3-9. One-dimensional lattice wave for \( k = -7\pi/4a \).

Fig. 3-10. Two waves separated by \( \Delta k = 2\pi/a \).

This wave is moving to the left \((k < 0)\) whereas... this wave is moving to the right \((k > 0)\). Is the motion of the atoms the same?