Materials 100B Homework #5 - Solution Set

**Problem 1**
Dislocations must have a Burgers vector that connects one crystal lattice site to another. There are therefore many possible magnitudes and directions for \( \mathbf{b} \). However, we observe (by experiment) that \( \mathbf{b} \) always corresponds in direction to a close packed row of atoms and in magnitude to the distance between lattice sites (atoms) along such a close packed row.

a.) Draw the atomic arrangements near an edge dislocation in a simple cubic crystal if that dislocation has a \( \mathbf{b} \) that is twice as long as the shortest \( \mathbf{b} \).

*The smallest \( \mathbf{b} \) will occur when one plane of atoms is inserted along the closest packed plane. To create a dislocation twice that length, two atom planes could be inserted yielding a \( \mathbf{b} \) twice the magnitude of the shortest \( \mathbf{b} \).*

![Diagram of atomic arrangements near an edge dislocation](image)

b.) By considering the energy per unit length of a dislocation with a longer \( \mathbf{b} \), rationalize why only the shortest possible \( \mathbf{b} \) is observed in practice.

*The energy of a dislocation is given by \( U = G*\mathbf{b}^2 \). The energy of one dislocation with \( \mathbf{b} = 2a \) is thus \( 4G*a^2 \) but if that dislocation “splits” into two separate dislocations each with \( \mathbf{b} = a \) their total energy will be \( 2G*a^2 \). Since nature adopts the lowest energy system possible, only dislocations with the shortest \( \mathbf{b} (=a) \) will be observed.*

c.) What will the Miller indices of dislocation \( \mathbf{b} \) directions in a BCC crystal of Ta?

*BCC crystals have Burgers vectors along the closest packed directions which are \(<111>\) type, i.e the body diagonals of the unit cell.*

d.) What will be the magnitude of \( \mathbf{b} \) in Ta? (use periodic table)
By definition in a BCC system, the atoms along the closest packed direction are “touching”. Therefore, the smallest $b$ possible is twice the atomic radius of Ta, which is $1.43 \, \text{Å}$. So the magnitude of the smallest $b$ is $2.86 \, \text{Å}$.

**Problem 2**

Copper has a fcc crystal structure and dislocations in Cu have Burgers vectors with magnitudes of $0.255 \, \text{nm}$. The slip plane in Cu is a (111) plane. A dislocation loop with a radius of $5000 \, \text{nm}$ lies in this plane with its Burgers vector in the (111) plane. If no other forces act on the dislocation other than the forces due to the applied shear stress $\tau$ (acting on the (111) plane in the direction of the Burgers vector) determine the magnitude of $\tau$. (Hint: you may also want to use the Periodic Table web site for this problem.)

Ans: $2.31 \, \text{MPa}$

In order to calculate the applied shear stress, we need to analyze the force balance on the dislocation loop. The tensile force on the dislocation, which attempts to keep the dislocation line straight, is given by:

$$F = \tau b L$$

The force balance on the dislocation loop is shown below:

According to the figure:

$$F = 2T = \tau b L$$

Where $T$ (the tension) is given by:

$$T = G b^2$$

with $G$ the shear modulus for copper and $b$ the Burgers vector. We now solve for $\tau$:

$$\tau = \frac{2G b}{L} = \frac{G b}{r}$$

The shear modulus $G$, for copper is obtained from the periodic table web site. (see problem 3 below for the method to find $G$)

$$G_{\text{Cu}} = 45.2 \, \text{GPa}$$
Therefore $\tau = 45.2(0.225\text{nm})/5000\text{nm} = 0.002 \text{ GPa} = 2 \text{ MPa}$.

**Problem 3**

An aluminum polycrystal contains hard aluminum oxide particles 5 nm in diameter with an average center-to-center spacing of 80 nm.

a) Estimate the force per unit length on a dislocation in the fcc aluminum necessary to push it ("extrude it") through the array of particles.

**ANS:** *Because the particles are hard (meaning they will not shear), use the equation*

\[
\tau_{\text{crit}} = \frac{2Gb}{l}
\]

*where $G$ is the shear modulus, $b$ is the Burgers vector and $l$ is the distance between the particles.*

$G = \frac{E}{2(1+\nu)}$, where $E$ is Young's modulus and $\nu$ is Poisson's ratio

$E = 70 \text{ GPa}$ (from webelements.com)

$\nu = 0.35$ (from webelements.com)

*Therefore, $G = 26 \text{ GPa}$*

$b =$ distance between lattice sites (atoms) along a close packed direction (from Problem 1)

Radius of an Al atom = 0.125 nm (from webelements.com)

*Therefore, $b = 0.250 \text{ nm}$*

$l =$ center-to-center distance minus two particle radii = $80\text{nm} - 2*2.5\text{nm} = 75 \text{ nm}$

\[
\tau_{\text{crit}} = \frac{(2*26 \text{ GPa}*0.250\text{nm})}{75 \text{ nm}} = 0.17 \text{ GPa}
\]

*To find the force per unit length,*

\[
F_{\text{crit}} = \tau_{\text{crit}}b
\]

\[
F_{\text{crit}} = 0.17 \text{ GPa} * 0.250 \text{ nm} = \underline{0.043 \text{ N/m}}
\]

b) Estimate the increase in the tensile yield stress due to these particles.

**ANS:**

$\sigma_y = 2\tau_{\text{crit}}$ (because it's in the book, $\sigma_y = 3\tau_{\text{crit}}$ also acceptable)

*From part a), $\tau_{\text{crit}} = 0.17 \text{ GPa}$*

*Therefore, $\sigma_y$ will increase by $2*0.17 \text{ GPa} = 0.34 \text{ GPa} = 345 \text{ MPa}$ (0.510 GPa increase if you use $\sigma_y = 3\tau_{\text{crit}}$)*

*We are not concerned with the level of detail that the book presents when it states the relation $\sigma_y = 3\tau_{\text{crit}}$. The point here is that $\sigma_y = (2\text{ to }3)\tau_{\text{crit}}$ where $\tau_{\text{crit}}$ is given approximately by $2Gb/l$.***
c) An annealing treatment at high temperatures causes the particles to grow in size and
decrease in number until they are 20 nm in diameter and are spaced 200 nm apart.
Estimate the increase or decrease in the tensile yield stress caused by this annealing
treatment.

ANS:
\[ l = \text{center-to-center distance minus two radii} = 200 \text{nm} - 2 \times 10 \text{nm} = 180 \text{nm} \]
\[ \tau_{\text{crit}} = \frac{(2 \times 26 \text{ GPa} \times 0.250 \text{nm})}{180 \text{ nm}} = 0.072 \text{ GPa} \]

Therefore, the change of \( \tau_{\text{crit}} \) is \( \Delta \tau_{\text{crit}} = -(0.17 \text{ GPa} - 0.072 \text{ GPa}) = -0.102 \text{ GPa} \)
From this change, \( \sigma_y \) decreases by \( 2 \times \Delta \tau_{\text{crit}} = 2 \times 0.102 \text{ GPa} = 0.204 \text{ GPa} \)
\( \sigma_y \) decreases by 0.204 GPa because of the particle growth and coarsening.

Problem 4
A step 50 b high is observed on one surface of square cross-section zinc crystal after it has been
plastically deformed in shear. Your friend claims that the step was caused by the motion of screw
dislocations that have moved in a direction perpendicular to the surface (see schematic drawing
below). Defend or refute your friend's claim.

\[ \text{Ans: This type of motion occurs from an edge dislocation, not a screw dislocation. A} \]
\[ \text{screw dislocation has a Burgers vector that is parallel to the dislocation line so if the} \]
\[ \text{dislocation moves as pictured, the step should be on the sides perpendicular to the one} \]
\[ \text{shown.} \]
Problem 5
From the phase diagram answer the following questions:

a.) Give compositions of the phases under the following conditions:

(see phase diagram above)

i.) 40°C and 40 wt% sugar - At this point we are still in the soluble region, so we have a one phase liquid solution 40wt% sugar, 60wt% water

ii) 20°C and 70 wt% sugar - We have passed the solubility limit into the 2 phase region so we will have two phases: a liquid solution composed of 62wt% sugar 38wt% water and a solid phase consisting of pure sugar.

iii.) 90°C and 70 wt% sugar - At this point we have one phase, a liquid solution consisting of 70wt% sugar 30wt% water.

iv.) 90°C and 100 wt% sugar - From the phase diagram we see that we will have pure solid sugar. This conclusion should also be obvious since we cannot have a solution with 100% of one component and 0% of another.

b.) Give a composition and starting temperature of a liquid syrup that will give maximum solid sugar output by thermal treatment.

Ans: We want to maximize the amount of sugar in the syrup while still retaining a one phase solution. This can be done by making a solution with a
composition at the solubility limit for the defined temperature. We can see from the phase diagram that solubility increases with temperature so we want to maximize our temperature.

Therefore, our formulation is 80% sugar at 100°C

c.) Specify a final temperature of the thermal treatment.

Ans: We want to yield the highest amount of solid sugar from our treatment. Since we know from the phase diagram that solubility decreases with temperature we would cool the solution down to the lowest possible temperature without freezing the water, just above 0°C.

d.) Starting with 100 kg of syrup how many kg of solid sugar does this treatment yield?

Ans: We must use the lever rule. Our original solution was 80% sugar by weight, however, at 0°C the solubility of sugar has dropped to 61% by weight. The length of the total lever arm is 39 while the length opposite the solid sugar (100wt%) is (80-61) = 19. So the weight percent sugar is equal to (19/39)*100 = 49wt%. Therefore 100 kg of starting solution will yield 49 kg of sugar.
Problem 6

The crystal structure of pure cobalt is fcc above 422 °C and hexagonal close packed below 422 °C. (Ignore the dashed dot line labeled "Magnetic Transformation" - this corresponds to when the $\alpha$-cobalt phase becomes ferromagnetic). The $\alpha$-cobalt solid solution has the fcc crystal structure while the (Cu) solid solution also has an fcc crystal structure (but with a different lattice parameter).

a. There are three unlabeled regions of two phase coexistence on the Co-Cu phase diagram. Identify these regions on the phase diagram and specify what two phases coexist in each.

b. On a two-component phase diagram, there can be a single temperature at which three phases coexist. If there is such a temperature on the Co-Cu phase diagram, identify it and tell the three phases that coexist and their compositions.

*Ans: At this point, the L, $\alpha$ and $\beta$ regions exist. 13% $\alpha$ Co, 92% L; 13% $\alpha$ Co, 95 $\beta$ Cu.*

c. For the following temperatures and overall compositions, identify the phase (or phases) that will be present at equilibrium. If more than one phase coexist, give the weight of each phase present in a 100 g sample as well as the compositions of each phase.
i. \( T = 1300 \, ^\circ \text{C} \) and 15 wt\% Cu

Ans: \( \alpha \) (Co) 15wt\% Cu.

ii. \( T = 400 \, ^\circ \text{C} \) and 15 wt\% Cu

Ans: \( \varepsilon \) (Co)[\(~0 \, \text{wt\% Cu}\)] and \( \beta \) (Cu) [\(~100 \, \text{wt\% Cu}\)]. Use lever rule to find weight of \( \beta \) (Cu) = \( 100 \times (15-0)/(100-0) = 15 \) g \( \beta \) (Cu); the rest is \( \varepsilon \) (Co).

iii. \( T = 900 \, ^\circ \text{C} \) and 90 wt\% Cu

Ans: \( \alpha \) (Co)[\(~4 \, \text{wt\% Cu}\)] and \( \beta \) (Cu)[\(~98 \, \text{wt\% Cu}\)]. Use lever rule to find weight of \( \beta \) (Cu) = \( 100 \times (90-4)/(98-4) = 91 \) g \( \beta \) (Cu); the rest is \( \alpha \) (Co).

iv. \( T = 1114 \, ^\circ \text{C} \) and 50 wt\% Cu

Ans: \( \alpha \) (Co)[\(~14 \, \text{wt\% Cu}\)] and L[\(~95 \, \text{wt\% Cu}\)]. Use lever rule to find weight of \( \beta \) (Cu) = \( 100 \times (50-14)/(95-14) = 44 \) g liquid; the rest is \( \alpha \) (Co).

v. \( T = 1110 \, ^\circ \text{C} \) and 50 wt\% Cu

Ans: \( \alpha \) (Co)[\(~14 \, \text{wt\% Cu}\)] and \( \beta \) (Cu)[\(~92 \, \text{wt\% Cu}\)]. Use lever rule to find weight of \( \beta \) (Cu) = \( 100 \times (50-14)/(92-14) = 46 \) g \( \beta \) (Cu); the rest is \( \alpha \) (Co).

d. What is the maximum amount of Cu that can be dissolved in the \( \alpha \)-cobalt phase? What temperature is needed to dissolve that much?

Ans: 19.7\%, at 1367 degrees C. (see phase diagram)