1. Solve the following problems from Chapter 18 of McQuarrie’s text (attached) related to kinetic theory of gases: 18-4,18-28.

2. Solve the following problems from Chapter 20 of McQuarrie’s text (attached) related to Brownian motion: 20-4, 20-18, 20-26.

3. Analyze the linear approach to equilibrium of the Chandrasekhar equation from the theory of Brownian motion. Do this by the method we followed in class to study the Boltzmann equation, i.e. assume

\[ f(r, p, t) = f_M(p)[1 + \psi(p, t) \exp(ik \cdot r)] \]

and analyze the resulting eigenvalue problem to \(O(k^2)\). Compare your results with those obtained for a dilute gas based on the analysis of the Boltzmann equation.

4. Consider a spherical macromolecule undergoing rotational Brownian motion in the high friction limit. If we denote the orientation of the transition dipole in the molecule at time \(t\) by a unit vector \(u(t)\), then we can envision the end of the vector as executing a Brownian trajectory restricted to the surface of the unit sphere as the molecule rotates in its solvent environment. The dipole orientation can be represented in spherical coordinates according to \(u = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)\) and the Fokker-Planck equation for the probability distribution function \(f(u, t)\) of the (high-friction) rotational diffusion process can be written

\[ \frac{\partial}{\partial t} f(u, t) = D_r \nabla_u^2 f(u, t) \]

where \(D_r\) is the rotational diffusion coefficient (units of frequency) and \(\nabla_u^2\) is the rotational diffusion operator:

\[ \nabla_u^2 = \frac{1}{\sin^2 \theta} \left[ \sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{\partial^2}{\partial \phi^2} \right] \]

(a) Develop an expansion of the fundamental (Green’s function) solution of this Fokker-Planck equation in spherical harmonic functions. Use this expansion to compute expressions for the fluorescence intensities \(I_{||}(t)\), \(I_{\perp}(t)\), and the anisotropy \(A(t)\) observed in a fluorescence depolarization experiment.
(b) From hydrodynamics and an Einstein formula for rotational diffusion, a rotating sphere of radius $a$ embedded in an incompressible fluid of viscosity $\eta$ (and assuming no-slip boundary conditions) has a rotational diffusion coefficient given by

$$D_r = \frac{k_B T}{8\pi \eta a^5}$$

Explain how you would use this result to extract information about the size of the macromolecule from a fluorescence depolarization experiment. What timescales should you worry about in designing the experiment?