1. Work through “demo10.nb” on our class website, substituting your own examples where appropriate.

2. Consider the matrix

\[ A = \begin{pmatrix} 5 & 8 & 2 & 0 \\ 0 & 9 & 1 & 4 \\ 8 & 6 & 4 & 2 \\ 1 & 9 & 5 & 3 \end{pmatrix} \]

(a). Using Mathematica, find the eigenvectors and eigenvalues of \( A \).
(b). Identify a basis for \( \mathbb{R}^4 \) spanned by the eigenvectors of \( A \).
(c). Find a \( 4 \times 4 \) matrix \( B \) such that \( B^{-1}AB \) is diagonal. Prove this by explicit calculation.

3. The following characteristic equation can be derived from a problem in the elasticity theory of thin plates:

\[
\begin{vmatrix}
1 & 1 & 0 & 1 \\
1 & -1 & 1 & 0 \\
\exp(\lambda) & \exp(-\lambda) & \sin(\lambda) & \cos(\lambda) \\
\exp(\lambda) & -\exp(-\lambda) & \cos(\lambda) & -\sin(\lambda)
\end{vmatrix} = 0
\]

Use Mathematica to find the two smallest eigenvalues (smallest positive values of \( \lambda \) that satisfy this equation).